

# Quasinormal modes and hidden conformal symmetry in the Reissner-Nordström black hole

Yong-Wan Kim <sup>1,a</sup>, Yun Soo Myung<sup>1,b</sup>, and Young-Jai Park<sup>2,3,c</sup>

<sup>1</sup>Institute of Basic Science and School of Computer Aided Science,  
Inje University, Gimhae 621-749, Korea

<sup>2</sup>Department of Physics and Center for Quantum Spacetime,  
Sogang University, Seoul 121-742, Korea

<sup>3</sup>Department of Global Service Management,  
Sogang University, Seoul 121-742, Korea

## Abstract

It is shown that the scalar wave equation in the near-horizon limit respects a hidden  $SL(2, \mathbb{R})$  invariance in the Reissner-Nordström (RN) black hole spacetimes. We use the  $SL(2, \mathbb{R})$  symmetry to determine algebraically the purely imaginary quasinormal frequencies of the RN black hole. We confirm that these are exactly quasinormal modes of scalar perturbation around the near-extremal black hole.

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<sup>a</sup>ywkim65@gmail.com

<sup>b</sup>ysmyung@inje.ac.kr

<sup>c</sup>yjpark@sogang.ac.kr

# 1 Introduction

It was known that the scalar wave equation in the low energy limit enjoys a hidden conformal symmetry in the non-extremal Kerr black hole which is not an underlying symmetry of the spacetime itself [1, 2]. The existence of such a hidden symmetry originates from the observation that scattering amplitudes of scalar off a black hole are given in terms of hypergeometric functions [3, 4] which form representations of the conformal group  $SL(2, R)$ . Importantly, this led to the conjecture that the non-extremal Kerr black hole with angular momentum  $J$  is dual to a  $CFT_2$  with the central charges  $c_L = c_R = 12J$  [1], which provides exactly the Bekenstein-Hawking entropy of the Kerr black hole. It is also found that the low energy scalar-Kerr scattering amplitudes coincide with thermal correlators of a  $CFT_2$ .

Recently, Chen and Long [5] have shown that one can use the hidden conformal symmetry to algebraically determine quasinormal mode spectrums as descendants of a lowest weight state in black hole spacetimes. On the other hand, the spin-2 and spin-3 quasinormal modes and frequencies around the BTZ black hole were constructed by the purely operator approach without any approximation [6, 7, 8].

Very recently, the authors [9] have shown that the scalar wave equation in the low energy limit enjoys a hidden  $SL(2, R)$  invariance in the Schwarzschild geometry. They have used the  $SL(2, R)$  symmetry to determine algebraically the quasinormal frequencies (QNFs) of the Schwarzschild black hole, and also shown that this yields the imaginary QNFs describing large damping. Explicitly, starting with the highest weight state  $\Phi^{(0)}$  with  $L_0\Phi^{(0)} = h\Phi^{(0)}$  and  $L_1\Phi^{(0)} = 0$ , all quasinormal modes could be constructed as descendants of  $\Phi^{(n)} = (L_{-1})^n\Phi^{(0)}$  obtained by acting with  $L_{-1}$  on the highest weight state  $\Phi^{(0)}$ . Then, one can read off the QNFs from the descendants.

We would like to mention that the method developed for the Kerr/ $CFT$  correspondence could not be directly applied to the RN black hole because there is no apparent  $AdS_2$  structure in the near-horizon geometry of the non-extremal RN black hole. In this direction, a hidden conformal symmetry could be extracted by making a 5D uplifted RN black hole [10]. However, this is not a genuine conformal symmetry developing in the four dimensional RN black hole. Recently, the authors [11] have found a hidden  $SL(2, R)$  symmetry in the near-horizon geometry of the RN black hole.

In this work, we will use the  $SL(2, R)$  symmetry to determine QNFs of the RN black hole algebraically. We confirm that these QNFs could be found exactly from the scalar perturbation around the near-extremal RN black hole [12]. We will also discuss QNFs comparing with potential pictures.

## 2 Hidden conformal symmetry

First, let us introduce the RN black hole whose metric is given by

$$ds_{\text{RN}}^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \quad (1)$$

with the metric function

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \quad (2)$$

Here,  $M$  and  $Q$  are the ADM mass and the electric charge of the RN black hole, respectively. Then, the inner ( $r_-$ ) and the outer ( $r_+$ ) horizons are obtained as

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2} \equiv M \pm r_0, \quad (3)$$

which satisfy  $f(r_{\pm}) = 0$ . Here  $r_0$  is a non-extremal parameter but a very small  $r_0 \ll M (\sim Q)$  corresponds to the near-extremal RN black hole. Note that we have an extremal RN black hole for  $r_0 = 0$ .

For the RN black hole, the relevant thermodynamic quantities are the Bekenstein-Hawking entropy and Hawking temperature

$$S_{BH} = \pi r_+^2, \quad (4)$$

$$T_H = \frac{r_+ - r_-}{4\pi r_+^2} = \frac{r_0}{2\pi r_+^2}, \quad (5)$$

respectively. Note that the surface gravity is defined as

$$\kappa = \frac{r_0}{r_+^2} = 2\pi T_H. \quad (6)$$

Now, let us consider a minimally coupled massless scalar propagating in the spacetimes (1), which satisfies the Klein-Gordon equation

$$\bar{\square}_{\text{RN}}\Phi = 0. \quad (7)$$

Using the Ansatz

$$\Phi(t, r, \theta, \phi) = e^{-i\omega t} \frac{R(r)}{r} Y_m^l(\theta, \phi), \quad (8)$$

together with the eigenvalue equation on  $S^2$

$$\begin{aligned} \Delta_{S^2} Y_m^l(\theta, \phi) &= \frac{1}{\sin\theta} \partial_{\theta}(\sin\theta \partial_{\theta} Y_m^l(\theta, \phi)) + \frac{1}{\sin^2\theta} \partial_{\phi}^2 Y_m^l(\theta, \phi) \\ &= -l(l+1) Y_m^l(\theta, \phi), \end{aligned} \quad (9)$$

the Klein-Gordon equation (7) transforms into the Schrödinger equation

$$\frac{d^2}{dr_*^2}R(r) + \left[\omega^2 - V_{\text{RN}}(r)\right]R(r) = 0, \quad (10)$$

where the tortoise coordinate is defined by  $dr_* = dr/f(r)$ , and the potential is given by

$$V_{\text{RN}}(r) = f(r) \left[ \frac{l(l+1)}{r^2} + \frac{2M}{r^3} - \frac{2Q^2}{r^4} \right]. \quad (11)$$

Next, we consider a coordinate transformation of

$$\rho \equiv -\frac{1}{2\kappa} \ln \left[ 1 - \frac{2r_0}{r - r_-} \right]. \quad (12)$$

In terms of  $\rho$ , the event horizon  $r = r_+$  is mapped into  $\rho \rightarrow \infty$ , while the spatial infinity  $r \rightarrow \infty$  into  $\rho \rightarrow 0$ :  $r \in [r_+, \infty]$  is mapped to  $\rho \in [\infty, 0]$ . In this work, we will only consider outside the event horizon because we wish to compute quasinormal modes. For the interior of the Cauchy horizon, see Ref. [11].

Using the new coordinate (12), the RN metric [11, 13] becomes

$$ds_\rho^2 = -\tilde{f}(\rho)dt^2 + \tilde{f}^{-1}(\rho) \left( \frac{r_0}{\sinh(\kappa\rho)} \right)^2 \left[ \left( \frac{\kappa}{\sinh(\kappa\rho)} \right)^2 d\rho^2 + d\theta^2 + \sin^2\theta d\phi^2 \right] \quad (13)$$

with

$$\tilde{f}(\rho) = \frac{1}{\left( e^{\kappa\rho} + \frac{r_-}{r_0} \sinh(\kappa\rho) \right)^2}. \quad (14)$$

Here, we note a useful relation between  $r$  and  $\rho$

$$r^2 = \tilde{f}^{-1}(\rho) \left( \frac{r_0}{\sinh(\kappa\rho)} \right)^2. \quad (15)$$

Then, let us consider a minimally coupled massless scalar propagating in the spacetimes (13), which satisfies the Klein-Gordon equation

$$\bar{\square}_\rho \Phi = 0. \quad (16)$$

Using the Ansatz

$$\Phi(t, \rho, \theta, \phi) = e^{-i\omega t} R(\rho) Y_m^l(\theta, \phi), \quad (17)$$

the Klein-Gordon equation (16) transforms into the second-order differential equation expressed in terms of  $\rho$

$$\left(\frac{\sinh(\kappa\rho)}{\kappa}\right)^2 \frac{d^2}{d\rho^2} R(\rho) + \left[ \frac{\omega^2}{\tilde{f}^2(\rho)} \left(\frac{r_0}{\sinh(\kappa\rho)}\right)^2 - l(l+1) \right] R(\rho) = 0, \quad (18)$$

which is again transformed into the Schrödinger-type equation [14]

$$\frac{d^2}{d\rho^2} R(\rho) + [\omega^2 - V_\omega(\rho)] R(\rho) = 0. \quad (19)$$

Here the  $\omega$ -dependent potential is given by

$$V_\omega(\rho) = \omega^2 \left[ 1 - \frac{(\kappa r_0)^2}{\tilde{f}^2(\rho) \sinh^4(\kappa\rho)} \right] + \frac{l(l+1)\kappa^2}{\sinh^2(\kappa\rho)}. \quad (20)$$

Next, we introduce three vector fields to develop a hidden conformal symmetry

$$\begin{aligned} L_1 &= \frac{1}{\kappa} e^{\kappa t} \left[ \cosh(\kappa\rho) \partial_t + \sinh(\kappa\rho) \partial_\rho \right], \\ L_0 &= -\frac{1}{\kappa} \partial_t, \\ L_{-1} &= \frac{1}{\kappa} e^{-\kappa t} \left[ \cosh(\kappa\rho) \partial_t - \sinh(\kappa\rho) \partial_\rho \right], \end{aligned} \quad (21)$$

which are slightly different from the previous construction [11]. These satisfy the  $\text{SL}(2, \mathbb{R})$  commutation relations

$$[L_0, L_{\pm 1}] = \mp L_{\pm 1}, \quad [L_1, L_{-1}] = 2L_0. \quad (22)$$

Then, the  $\text{SL}(2, \mathbb{R})$  Casimir operator is constructed by

$$\begin{aligned} \mathcal{H}^2 &= L_0^2 - \frac{1}{2}(L_1 L_{-1} + L_{-1} L_1) \\ &= -\left(\frac{\sinh(\kappa\rho)}{\kappa}\right)^2 \partial_t^2 + \left(\frac{\sinh(\kappa\rho)}{\kappa}\right)^2 \partial_\rho^2. \end{aligned} \quad (23)$$

We approximate the second term in Eq. (18) as

$$\frac{\omega^2}{\tilde{f}^2(\rho)} \left(\frac{r_0}{\sinh(\kappa\rho)}\right)^2 = \omega^2 r^4 \left(\frac{\sinh(\kappa\rho)}{r_0}\right)^2 \quad (24)$$

$$\approx \omega^2 (r_+ + 2r_0 e^{-2\kappa\rho})^4 \left(\frac{\sinh(\kappa\rho)}{r_0}\right)^2 \quad (25)$$

$$\approx \omega^2 r_+^4 \left(\frac{\sinh(\kappa\rho)}{r_0}\right)^2 = \omega^2 \left(\frac{\sinh(\kappa\rho)}{\kappa}\right)^2, \quad (26)$$

where we use Eq. (15) in deriving (24) and use  $r = r_+ + 2r_0e^{-2\kappa\rho}$  for the near-horizon approximation ( $r \rightarrow r_+$ , or  $\rho \rightarrow \infty$ ) to derive (25). Finally, we obtain (26) when employing the low-energy approximation [9]. In the near-horizon and low-energy approximations which are necessary to develop the hidden conformal symmetry, the first two terms of the  $\omega$ -dependent potential (20) disappear, leading to the last term only.

As a result, comparing (18) with (23), the Klein-Gordon equation in the near-horizon and low-energy approximations can be rewritten in terms of the  $\text{SL}(2, \mathbb{R})$  Casimir operator  $\mathcal{H}^2$  as

$$\bar{\square}_\rho \Phi = 0 \rightarrow \mathcal{H}^2 \Phi = l(l+1)\Phi. \quad (27)$$

The latter can be rewritten as the Schrödinger equation of

$$\frac{d^2}{d\rho^2} R(\rho) + \left[ E - V_{\text{HCS}}(\rho) \right] R(\rho) = 0, \quad (28)$$

where the energy  $E$  is

$$E = \omega^2, \quad (29)$$

and the HCS-potential takes the form

$$V_{\text{HCS}}(\rho) = \frac{l(l+1)\kappa^2}{\sinh^2(\kappa\rho)}. \quad (30)$$

Therefore, the massless scalar wave equation carries the hidden conformal symmetry which is not a spacetime symmetry. We note that the hidden conformal symmetry is realized only when approximating the  $\omega$ -dependent potential  $V_\omega(\rho)$  (20) by the HCS-potential  $V_{\text{HCS}}(\rho)$  (30) in the Schrödinger equation.

### 3 Quasinormal modes constructed by operator method

Now, let us use the hidden conformal symmetry to derive QNFs of the RN black hole. First, we define the primary state by  $\Phi^{(0)}$  which satisfies

$$L_0 \Phi^{(0)} = h \Phi^{(0)}, \quad (31)$$

and the highest weight condition

$$L_1 \Phi^{(0)} = 0. \quad (32)$$

Then, since  $\Phi^{(0)}$  takes the form

$$\Phi^{(0)} = e^{-i\omega_0 t} R^{(0)}(\rho) Y_m^l(\theta, \phi), \quad (33)$$

one has a conformal weight

$$h = i \frac{\omega_0}{\kappa} = i \frac{\omega_0}{2\pi T_H}. \quad (34)$$

On the other hand, for  $\Phi^{(0)}$ , the  $SL(2, \mathbb{R})$  Casimir operator satisfies

$$\mathcal{H}^2 \Phi^{(0)} = h(h+1) \Phi^{(0)}. \quad (35)$$

Comparing Eq. (35) with Eq. (27), one has

$$h = \frac{1}{2} [1 \pm (2l+1)]. \quad (36)$$

Then, together with Eq. (34), one can find

$$\omega_0 = -i \frac{\kappa}{2} [1 \pm (2l+1)]. \quad (37)$$

Since the QNFs are purely imaginary  $\omega_I < 0$  ( $\omega = \omega_R + i\omega_I$ ) with  $\omega_R = 0$ , we choose the upper sign as

$$\omega_0 = -i\kappa(l+1). \quad (38)$$

Next, all the descendants are constructed by

$$\Phi^{(n)} = (L_{-1})^n \Phi^{(0)} \quad (39)$$

so that we have

$$\Phi^{(n)} = e^{-i\omega_n t} R^{(n)}(\rho) Y_m^l(\theta, \phi), \quad (40)$$

where the QNFs are read off as

$$\omega_n = \omega_0 - i\kappa n = -i\kappa [n + l + 1], \quad (41)$$

which is our main result.

Note that the RN black hole has the same conformal symmetry of the Schwarzschild black hole [9]. We observe that the QNFs in (41) have the leading term of  $-i2\pi T_H$ , while they do not have subleading terms.

Moreover, the  $n$ -th radial eigenfunction  $R^{(n)}(\rho)$  takes the form

$$\begin{aligned} R^{(n)}(\rho) &= (\kappa)^{-n} \left( -i\omega_{n-1} \cosh(\kappa\rho) - \sinh(\kappa\rho) \frac{d}{d\rho} \right) \\ &\quad \times \left( -i\omega_{n-2} \cosh(\kappa\rho) - \sinh(\kappa\rho) \frac{d}{d\rho} \right) \\ &\quad \cdots \times \left( -i\omega_0 \cosh(\kappa\rho) - \sinh(\kappa\rho) \frac{d}{d\rho} \right) R^{(0)}(\rho). \end{aligned} \quad (42)$$

We also have

$$L_0 \Phi^{(n)} = (h + n) \Phi^{(n)}, \quad (43)$$

which implies that  $\Phi^{(n)}$  forms a principal discrete highest weight representation of the  $\text{SL}(2, \mathbb{R})$ .

Now, we wish to solve the highest weight condition (32) to determine the highest weight state  $R^{(0)}(\rho)$

$$\left[ -i\omega_0 \cosh(\kappa\rho) + \sinh(\kappa\rho) \frac{d}{d\rho} \right] R^{(0)}(\rho) = 0. \quad (44)$$

The solution is given by

$$R^{(0)}(\rho) = C \left[ \sinh(\kappa\rho) \right]^{i \frac{\omega_0}{\kappa}}. \quad (45)$$

Here we note that the tortoise coordinate  $r_*$  given by

$$r_* = r - \frac{r_-^2}{2r_0} \ln(r - r_-) + \frac{1}{2\kappa} \ln(r - r_+) \quad (46)$$

approaches

$$r - r_+ \sim e^{2\kappa r_*}, \quad (47)$$

as  $r \rightarrow r_+$ .

On the other hand,  $\rho$ -coordinate (12) goes to

$$r - r_+ \sim e^{-2\kappa\rho} \quad (48)$$

in the near-horizon region so that  $\rho$  behaves as

$$\rho \sim -r_* \quad (49)$$

in this region. This gives us the solution (45) which behaves as

$$R^{(0)} \sim e^{-i\omega_0 r_*}, \quad (50)$$



for  $r \rightarrow r_+$ . This is obviously the ingoing mode propagating into the horizon. For the  $n$ -th radial eigenfunction, one can easily show by induction

$$R^{(n)} \sim e^{-i\omega_n r_*}, \quad \text{as } r_* \rightarrow -\infty. \quad (51)$$

In order to obtain the complete QNFs, we have to impose the two boundary conditions: ingoing mode at the horizon and outgoing mode at spatial infinity. However, it seems that  $R^{(n)}(\rho)$  does not satisfy outgoing boundary condition at infinity because they are the solutions in the near-horizon and low-energy limits. In view of the Schrödinger equation in the whole space-times from  $\rho = 0$  to  $\infty$ , this amounts to approximating the  $\omega$ -dependent potential  $V_\omega(\rho)$  by the HCS-potential  $V_{\text{HCS}}(\rho)$ .

## 4 QNFs around near-extremal RN black hole

We know that the literature of quasinormal modes for the RN black hole is vast. It is unclear that the purely imaginary QNFs (41) capture what kind of RN black hole, even though they have obtained from the near-horizon and low-energy approximations of a scalar perturbation around the RN black hole. This requires to know the whole picture of a scalar perturbation around a specified RN black hole. We will show that *the specified RN black hole is exactly the near-extremal RN black hole*.

For this purpose, we briefly mention a way to obtain the QNFs of the near-extremal black hole which is based on the work of Chen et al. [12]. Then, we compare the previous results with the ones obtained from the near-extremal RN black hole. In the near-extremal limit ( $r_0 = \sqrt{M^2 - Q^2} \ll M \sim Q$ ) of

$$r \rightarrow Q + \tilde{\rho}, \quad M \rightarrow Q + r_0, \quad (52)$$

the RN metric (1) can be rewritten with  $r_0 = \sqrt{2Q(M - Q)}$

$$ds_{\text{NE}}^2 = -\frac{\tilde{\rho}^2 - r_0^2}{Q^2} dt^2 + \frac{Q^2}{\tilde{\rho}^2 - r_0^2} d\tilde{\rho}^2 + Q^2 d\Omega_2^2, \quad (53)$$

whose geometry is given by  $\text{AdS}_2 \times S^2$ . Here  $\tilde{\rho} \in [r_0, \infty]$ . For this near-extremal RN black hole, the surface gravity is given by

$$\tilde{\kappa} = \frac{r_0}{Q^2}. \quad (54)$$

Here, note that the surface gravity (54) is obtained directly from the near-extremal solution (53), while one can approximate the surface gravity (6) in the leading order to give

$$\kappa \approx \tilde{\kappa} \left[ 1 - \mathcal{O}\left(\frac{2r_0}{M}\right) \right]. \quad (55)$$

We observe that for the near-extremal black hole, its surface gravity is given by  $\tilde{\kappa}$ . Then, the Klein-Gordon equation (16) with the ansatz (17) can be written by

$$\frac{d}{d\tilde{\rho}} \left( (\tilde{\rho}^2 - r_0^2) \frac{d}{d\tilde{\rho}} \right) R(\tilde{\rho}) + \left( \frac{\omega^2 Q^4}{\tilde{\rho}^2 - r_0^2} - l(l+1) \right) R(\tilde{\rho}) = 0. \quad (56)$$

Introducing the tortoise coordinate defined by

$$\rho_* = \frac{1}{2\tilde{\kappa}} \ln \left( \frac{\tilde{\rho} + r_0}{\tilde{\rho} - r_0} \right), \quad \rho_* \in [\infty, 0], \quad (57)$$

the Klein-Gordon equation becomes the Schrödinger equation

$$\frac{d^2}{d\rho_*^2} R(\rho_*) + \left[ \omega^2 - V_{\text{NE}}(\tilde{\rho}) \right] R(\rho_*) = 0, \quad (58)$$

where the near-extremal RN potential is given by

$$V_{\text{NE}}(\tilde{\rho}) = \frac{l(l+1)(\tilde{\rho}^2 - r_0^2)}{Q^4}. \quad (59)$$

Moreover, solving the tortoise coordinate in terms of  $\tilde{\rho}$  as

$$\tilde{\rho} = r_0 \coth(\tilde{\kappa}\rho_*), \quad (60)$$

one can easily show that the near-extremal RN potential leads to

$$V_{\text{NE}}(\rho_*) = \frac{l(l+1)\tilde{\kappa}^2}{\sinh^2(\tilde{\kappa}\rho_*)}, \quad (61)$$

which is the exactly same form of  $V_{\text{HCS}}(\rho)$  in Eq. (30) when replacing  $\tilde{\kappa}$  and  $\rho_*$  by  $\kappa$  and  $\rho$ . Here  $\rho_*$  mimics exactly  $\rho$  for describing the region outside the event horizon. This means that our previous results is valid for the near-extremal black hole only where the surface gravity of the RN black hole  $\kappa$  is approximated by the surface gravity of the near-extremal RN black hole ( $\kappa \approx \tilde{\kappa}$ ).

On the other hand, as obtained in Ref. [12], the QNFs of a massive charged scalar with mass  $m$  and charge  $q$  around the near-extremal RN black hole is given by

$$\omega_n = -\tilde{\kappa}b - i\tilde{\kappa} \left( n + \frac{1}{2} \right), \quad (62)$$

where the parameter  $b$  is given by

$$b = \sqrt{(q^2 - m^2)Q^2 - \left( l + \frac{1}{2} \right)^2}. \quad (63)$$

In this work, since we are considering the uncharged massless scalar field, the parameter  $b$  takes the purely imaginary value

$$b = i \left( l + \frac{1}{2} \right). \quad (64)$$

As a result, one obtains the QNFs for the near-extremal RN black hole as

$$\omega_n = -i\tilde{\kappa}(n + l + 1), \quad (65)$$

which is the exactly same form of our result (41) up to the leading order. This states clearly that the QNFs of the scalar perturbation around the RN black hole with the hidden conformal symmetry are those obtained from the scalar perturbation around the near-extremal RN black hole.

At this stage, we mention that the angular momentum quantum number  $l$  is included as the real part of quasinormal frequencies

$$\omega_n^{\text{PT}} = \kappa \left[ \sqrt{l(l+1)} - \frac{1}{4} - i \left( n + \frac{1}{2} \right) \right]. \quad (66)$$

for the Pöschl-Teller approximation [15] of the RN potential (11) [16]. Its potential is given by

$$V_{\text{PT}}(\tilde{r}_*) = \frac{l(l+1)\kappa^2}{\cosh^2(\kappa\tilde{r}_*)} \quad (67)$$

for the tortoise coordinate  $\tilde{r}_* \in [-\infty, \infty]$ . However, as is shown in (41), we have  $l$  in the imaginary frequency. This could be explained by mentioning that our QNFs are those obtained from the scalar perturbation around the near-extremal RN black hole (53) whose geometry is  $\text{AdS}_2 \times S^2$ . This productive geometry describes effectively the two-dimensional black hole whose quasinormal frequencies are usually given by purely imaginary QNFs [17]. That is, the QNFs of the scalar perturbation around the near-extremal black hole are purely imaginary.

## 5 Potential Picture

Now, let us investigate the validity of the QNFs  $\omega_n$  in Eq. (41) by comparing the relevant potentials. In Fig.1, we plot these potentials with  $l = 10, r_0 = 1$  ( $r_+ = 2.1, r_- = 0.1, M = 1.1, Q = 0.46, \kappa = 0.23$ ) for  $V_{\text{RN}}(r)$ ,  $V_\omega(\rho)$  with  $\omega = 1$ ,  $V_{\text{HCS}}(\rho)$ , and  $V_{\text{NE}}(\rho_*)$ .

First, as is shown in Fig.1-(a),  $V_{\text{RN}}(r)$  is the potential barrier existing outside the event horizon at  $r = r_+$ . Because of its asymptotic behavior of

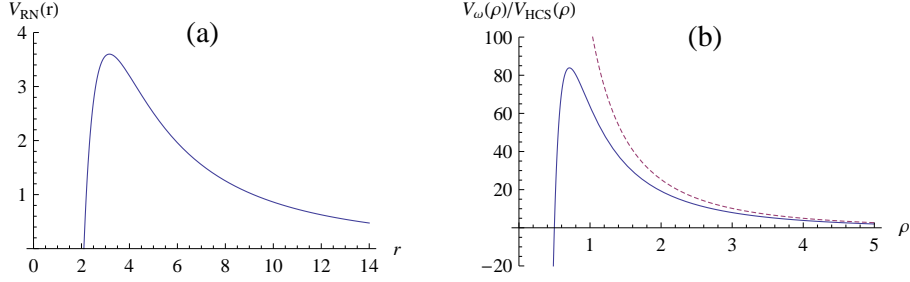


Figure 1: Potential pictures for  $l = 10$ : (a) the RN potential  $V_{\text{RN}}(r)$  with  $M = 1.10$ ,  $Q = 0.46$ , (b) the  $\omega$ -dependent potential  $V_{\omega}(\rho)$  in solid curve with  $\omega = 1$ ,  $r_0 = 1$ ,  $r_- = 0.10$ , and the HCS potential  $V_{\text{HCS}}(\rho)$  in dashed curve with  $r_0 = 1$ . We note that the potential  $V_{\text{NE}}(\rho_*)$  is the same graph as in  $V_{\text{HCS}}(\rho)$  when replacing  $\rho$  by  $\rho_*$ .

$V_{\text{RN}} \sim 1/r^2$  as  $r \rightarrow \infty$ , one could not find the analytic expression for the QNFs, in compared to the Pöschl-Teller potential  $V_{\text{PT}}(\tilde{r}_*)$  in (11).

Second, as depicted by a solid curve in Fig.1-(b), the  $\omega$ -dependent potential  $V_{\omega}(\rho)$  shows a negative potential around  $\rho = 0$  ( $r \rightarrow \infty$ ). At this stage, it seems that we do not understand the appearance of the negative potential which may induce the instability of the RN black hole. In order to understand the negativeness of the potential, we take its limit of  $\rho \rightarrow 0$  as

$$V_{\omega}^{\rho \rightarrow 0}(\rho) = \omega^2 - \omega^2 r_+^4 \left( \frac{1}{\rho} + \frac{r_+ + r_-}{2r_+^2} \right)^4 + \frac{l(l+1)}{\rho^2}, \quad (68)$$

where the second term is responsible for the negativeness in the solid curve in Fig.1-(b), while the last term makes the potential positively infinite as drawn by the dashed curve in Fig.1-(b). Hence, requiring the low energy limit of  $\omega \rightarrow 0$ , one neglects the second term in favor of the last term, effectively leading to the HCS-potential depicted as the dashed curve in Fig.1-(b).

Third, concerning for the  $V_{\text{HCS}}(\rho)$  potential, we have to say that this potential is not a genuine potential which is valid for the whole RN black hole but a form of the potential obtained by the approximation (26) to develop the hidden conformal symmetry near the event horizon. As was shown in section 4,  $V_{\text{HCS}}(\rho)$  is exactly the potential  $V_{\text{NE}}(r_*)$  in (61) of the scalar perturbation around the near-extremal RN black hole.

Finally, we mention that the graphs in Fig.1 are designed for a non-extremal RN black hole of  $r_+ = 2.1$ ,  $r_- = 0.1$ ,  $M = 1.1$ ,  $Q = 0.46$ ,  $\kappa = 0.23$ . Even for the near-extremal RN black hole, the behaviors of  $V_{\omega}(\rho)$  and  $V_{\text{HCS}}(\rho)$  are similar to those in Fig.1-(b) because  $\rho$ -coordinate was used to draw the pictures whose range is from  $\rho = \infty$  (event horizon) to  $\rho = 0(\infty)$ .

In conclusion, we have derived the imaginary QNFs (41) of the RN black hole by making use of the hidden conformal symmetry developed in the near-horizon and low-energy approximations of the massless Klein-Gordon equation. We see that the operator approach has a limitation because the  $\omega$ -dependent potential (20) is approximated by the HCS potential (30). This means that developing the hidden conformal symmetry in the near-horizon means losing the large  $r$  behavior of the potential in the whole RN black hole. Consequently, the imaginary QNFs (41) are the quasinormal frequencies of the scalar perturbation around the near-extremal RN black hole.

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